Final Exam

ZF15697 Calculus in Engineering, Spring 2021
School of BioMedical Convergence Engineering, PNU
June. 17. 11:00 - 14:00

I. REMARK

- You are permitted on three page of notes.
- There are a total of 100 points in the exam.
- You must SHOW YOUR WORK to get full credit.

II. PROBLEM SET

1) An apple is grown in PNU campus. The amount of weight change with time is given as

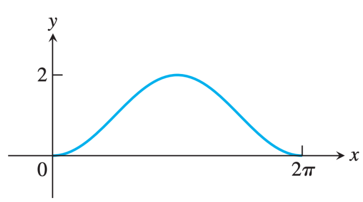
$$\frac{dy}{dt} = (1+y)(e^t t)$$

Assume that the weight y is 0 (kg) when $t = 0$. What is the weight when $t = \ln 2$.

2) The curve with parametric equations

$$x = t,\ y = 1 - \cos t,\ 0 \le t \le 2\pi$$

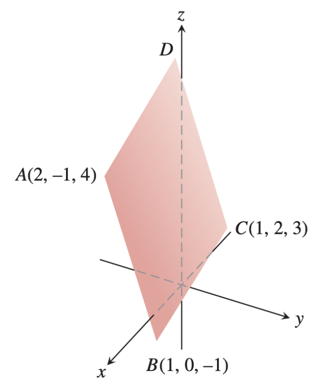
is called a sinusoid and is shown the accompanying figure. Find the point (x,y) where the slope of the tangent line is largest.



3) Solve the problem for $\mathbf{r}$ as a vector function of t,

Differential equation: $\frac{d^2\mathbf{r}}{dt^2} = -(\mathbf{i}+\mathbf{j}+\mathbf{k})$
Initial conditions: $\mathbf{r}(0) = 10\mathbf{i} + 10\mathbf{j} + 10\mathbf{k}$
and $\frac{d\mathbf{r}}{dt}\Big|_{t=0} = \mathbf{0}$

4) The parallelogram shown here has vertices at A(2,-1, 4), B(1, 0, -1), C(1,2,3), and D. Find
   a) the coordinates of D,



   b) the cosign of the interior angle at B,
   c) the vector projection of $\overrightarrow{BA}$ onto $\overrightarrow{BC}$,
   d) the area of the parallelogram,
   e) an equation for the plane of the parallelogram.

5) Suppose $\mathbf{r}(t) = (e^t \cos t)\mathbf{i} + (e^t \sin t)\mathbf{j}$. Show that the angle between $\mathbf{r}$ and $\mathbf{a}$ never changes. What is the angle?

6) Find $\mathbf{T}$, $\mathbf{B}$, $\mathbf{N}$, and κ as a function of t if

$$\mathbf{r}(t) = (\sin t)\mathbf{i} + (\sqrt{2}\cos t)\mathbf{j} + (\sin t)\mathbf{k}$$

7) Show that the curve

$$\mathbf{r}(t) = (\ln t)\mathbf{i} + (t \ln t)\mathbf{j} + (t)\mathbf{k}$$

is tangent to the surface

$$xz^2 - yz + \cos xy = 1$$

at $(0, 0, 1)$.

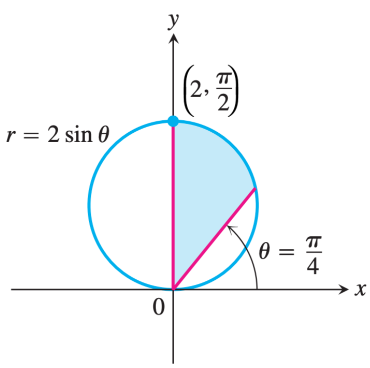
8) Find the derivative of $f(x,y,z) = xyz$ in the direction of the velocity vector of the helix

$$\mathbf{r}(t) = (\cos 3t)\mathbf{i} + (\sin 3t)\mathbf{j} + 3t\mathbf{k}$$

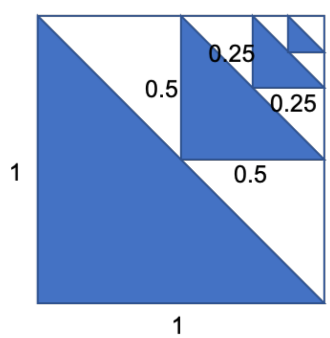
at $t = \pi/3$.

9) Answer the following questions.

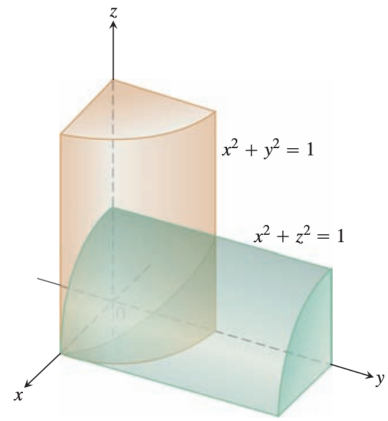
a) Find the area of the region bonded by circle $r = 2\sin\theta$ for $\pi/4 \le \theta \le \pi/2$.



b) Consider the infinite sequence of shaded right triangles in the accompanying diagram. Compute the total area of the triangles.



10) Find the volume of the region where the region common to the interiors of the cylinders $x^2 + y^2 = 1$ and $x^2 + z^2 = 1$, one-eighth of which is shown in the accompanying figure



III. Supporting Notes

- $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
- $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
- $\sin 2x = 2\sin x \cos x$